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On Quantum Field Brownian Motion, Decoherence
and Semiquantum Chaos ¹

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Abstract: Entropy production in quantum (field) systems requiring environment-induced decoherence is described in a Gaussian variational approximation. The new phenomenon of Semiquantum Chaos is reported.

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1. Introduction

Entropy is generated in complex systems by a dynamical separation of “relevant” degrees of freedom (observables) from “environment” modes, which we integrate out. We generalize the Feynman-Vernon influence functional for single-particle quantum Brownian motion to quantum field theory [1]. Our nonperturbative approach is based on a Gaussian variational principle, i.e. time-dependent Hartree-Fock approximation (TDHF) in terms of Wightman functions. We calculate for the first time the induced *von Neumann entropy* (vNE) in non-equilibrium situations [2]. The functional formalism is sufficiently general to study in field theory, e.g., phase transitions and environment-induced quantum decoherence effects [1, 3].³

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³For brevity we refer to the references, from where relevant literature can be traced.

Working on the quantum to classical transition in the long wavelength regime of nonlinear systems, we attempt to understand whether and to what extent classical deterministic chaos is important for properties of quantum fields. Presently, we report on the zero-dimensional infrared limit of a prototype ϕ^4 field theory, i.e. a quantum

mechanical double-well oscillator, which is regular in the classical limit. However, including TDHF quantum corrections leads to a new phenomenon: *Semiquantum Chaos* [4]. There exist energy dependent transitions between regular behavior and deterministic chaos in this model. Extensions of this work may lead towards experiments on semiquantum chaos and the underlying quantum decoherence effect.

2. Quantum Field Brownian Motion and Decoherence

The long-standing “entropy puzzle” [1] dates back to Fermi and Landau trying to understand the rapid thermalization of high energy density ($\gg 1 \text{ GeV/fm}^3$) matter in strong interactions. It can be related to the concepts of *open quantum systems* and *environment-induced quantum decoherence* [1, 3]. Effectively, *unitary time evolution* of the observed part of the system breaks down in the transition from a quantum mechanically pure initial state to an impure final state. Evolution by $\exp(-i\hat{H}t)$ preserves the purity of a state and, therefore, *cannot* produce entropy. A complex pure-state quantum system, however, can show quasi-classical behaviour, i.e. an impure density (sub)matrix together with decoherence of the associated “pointer” states in the observed subsystem [1, 3].

This has been studied in detail in a nonrelativistic single-particle model resembling an electron coupled to the quantized electromagnetic field, however, with an enhanced infrared spectral density. The Feynman-Vernon in-

fluence functional remarkably yields in the short-time strong-coupling limit that the quantum particle behaves like a *classical particle* [1]: Gaussian particle wave packets experience *friction* and *localization*, i.e. no quantum mechanical spreading, and their coherent *superpositions decohere*. Decoherence here leads directly to vNE, $S \equiv \text{Tr} \hat{\rho} \ln \hat{\rho}$, $\hat{\rho} \equiv \text{particle dens. m.}$ – In the related third of Refs. [1] we find exponentially fast entropy production in a quantum system which is chaotic classically. This requires a small decohering effect, e.g. due to vacuum fluctuations coupled in from a higher energy scale. Contrary to previous claims, we observe that a zero temperature environment inducing *partial decoherence* is sufficient.

In field theory one naturally considers two coupled fields instead, e.g. fast and slow modes of one self-interacting field, respectively. This amounts to an observable field interacting with an environment, i.e. *quantum field Brownian motion*. In the functional Schrödinger picture employing TDHF a Cornwall-Jackiw-Tomboulis type effective action and renormalizable equations of motion are derived [1]. With the related Gaussian wave functionals the non-equilibrium vNE for scalar fields follows, $S(t) = -\text{Tr}\{\ln(1 - Y) + Y(1 - Y)^{-1} \ln Y\}$, tracing over coordinates and Y explicitly involving two-point functions of both fields [2].

3. Semiquantum Chaos in Classically Regular Systems

Following the above approach to quantum field Brownian motion, it becomes desirable to apply and test it in simple cases. To study the semi-classical TDHF approximation, we consider time-dependent homogeneous configurations in a Higgs field theory [4]. This reduces to a quantum mechanical double-well oscillator, i.e. a closed system to begin with.

Including TDHF corrections into one-dimensional classical equations of

motion, e.g., which otherwise behave regularly according to the Poincaré-Bendixson theorem, introduces *additional nonlinear terms* together with an *additional effectively classical degree of freedom*, however, representing quantum fluctuations [4]. We obtain an autonomous first-order nonlinear flow system (four equations instead of classically two). Trajectories in the corresponding two-dimensional effective potential behave regularly near potential minima or for sufficiently high excitation energy and show deterministic chaos connected to barrier/tunneling effects at intermediate energies. We analyze this *semiquantum chaos* by Poincaré sections and a new frequency correlation function related to the density matrix of the system [4], which both reflect the coexistence of KAM tori and regions of irregularity. The phenomenon is not tied to this model.

Next, we have to include interactions with an environment. We expect (cf. Section 2) that tuned radiation drives the double-well oscillator, e.g., into the semiclassical regime. Thus, we necessarily go from a nondissipative system to an open one, in order to make semiquantum chaos a measurable effect. Presumably, it can be observed in experiments with irradiated solid state quantum dots.

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